STEREOMETRIC CONSTANTS OF THE BRONCHIAL TREE AND ARTERIES OF THE LUNG

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A study of five preparations of the lungs of adult dogs showed that the decrease in the mean diameters of the artery and bronchi from the roots of the lung to its periphery obeys the exponential equation $D = e^{-kz+b}$, where z is the numerical order of the branches, D the diameter of the artery or bronchus in branches of the z-th order, k the rate of change of the mean diameters, and b the logarithm of the diameter of the artery or bronchus for which $z = 0$. The parameter b was shown to determine the dimensions of corrosion preparations whereas the parameter k is independent of the true dimensions of the lung and its mean value is 0.25. The use of the parameter k and the area beneath the intervals of branching orders 1-7, 7-14, and 14-21 of the standardized function $D = e^{-kz}$ is recommended for topical diagnosis and investigation of the dynamics of lung diseases.

KEY WORDS: bronchial tree; arteries of the lung; stereometric constants.

The exponential character of changes in the diameters of branches of the bronchial tree and the arterial system from the roots of the lung to its periphery was demonstrated by anatomists many years ago [1, 2, 4] but it is only in the last decade that attempts have been made to describe the relationship between the numerical order of the branches and their mean diameter mathematically [6, 7]. If it is assumed that $D = e^{-kz+b}$ (where z is the order of the branches, D the diameter of the branches in the z-th order, k the rate of change of the diameters, and b the logarithm of diameter when $z = 0$) and if this equality is used in Rosen's equation [5] $(0.794)^z = Dz/Di$, we thus obtain for $z = 21$: $(0.794)^{21} = e^{-21k+b}/e^{-k+b} = e^{-20k}$; $k = -0.243$. Clearly the value of k for all normal arterial and bronchial preparations will be constant. In disease there is a redistribution of diameters [3]. The writers postulated that under these circumstances the value of k, characterizing the state of the lung, will also change.

The aims of the investigation were as follows: to prove that the rate of change of the mean diameters of bronchial and arterial branches from the roots of the lung to its periphery obeys the exponential equation $D = e^{-kz+b}$; to show that the rate of change of the mean diameters for the normal lung is in fact a constant and is independent of the size of the organ; to develop a typical model of distribution of branches of the bronchial tree and arterial system and to deduce standard parameters.

EXPERIMENTAL METHOD

Experiments were carried out on five dogs' lungs. The diameters of the branches of the 1st-9th orders were measured in corrosion preparations of the bronchial tree and arterial system obtained with the lung inflated to ¾ of the vital capacity. The mean diameters of the branches in each order were found by statistical analysis. The results were used to calculate the mean rate of change of the diameters k and the logarithm of the diameter of the bronchus or artery of zero order (b) from the equation $D = e^{-kz+b}$ by the method of least squares. To determine the individual features of the bronchi and arteries in different

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TABLE 1. Mean Diameters of Bronchi of 1st-9th Orders (in mm)

Specimen No.	Order of branches								
	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
1	12,67	10,08	7,40	5,46	4,48	3,29	2,72	2,59	1,70
2	10,27	9,35	7,24	4,95	3,85	3,25	2,45	1,90	1,35
3	9,02	7,62	7,32	4,96	4,00	3,25	2,25	1,85	1,41
4	12,75	10,05	8,15	5,95	4,15	3,45	2,25	1,75	1,45
5	10,43	8,08	6,18	4,75	3,83	2,97	2,21	1,68	1,25

TABLE 2. Mean Diameters of Arteries of 1st-9th Orders (in mm)

Specimen No.	Order of branches								
	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
1	10,23	7,16	5,19	3,83	3,42	2,50	2,29	1,63	1,26
2	8,17	7,28	4,79	4,06	3,12	2,40	1,92	1,46	1,15
3	9,28	5,48	4,33	3,37	3,02	2,38	2,17	1,57	1,07
4	10,26	9,03	6,50	4,15	3,35	2,40	1,85	1,45	1,15
5	7,87	6,29	4,53	3,75	2,93	2,18	1,61	1,23	0,93

TABLE 3. Differences between Theoretically and Experimentally Determined Mean Diameters of Branches of Bronchi and Arteries of 21st Order in the Lung

Specimen No.	Mean diameters (mm)				Deviations of theor. values of diameters from experiment (mm)	
	theoretical		experimental			
	arteries	bronchi	arteries	bronchi	arteries	bronchi
1	0,0550	0,1158	0,0619	0,1097	0,0069	0,0061
2	0,0821	0,0608	0,0764	0,0623	0,0057	0,0013
3	0,0612	0,0907	0,0635	0,0861	0,0023	0,0046
4	0,0723	0,0672	0,0699	0,0704	0,0024	0,0032
5	0,0498	0,0622	0,0490	0,0617	0,0008	0,0005

TABLE 4. Values of Parameters k and b for each Preparation of Bronchial Tree and Arterial System

Specimen No.	*k*		*b*	
	arteries	bronchi	arteries	bronchi
1	0,2513	0,2463	2,4902	2,7535
2	0,2452	0,2492	2,3428	2,5647
3	0,2492	0,2483	2,3124	2,5935
4	0,2469	0,2514	2,3542	2,6240
5	0,2594	0,2578	2,3290	2,5980

lungs and to obtain standard characteristics the areas were calculated for intervals 1-7, 7-14, and 14-21 of branching orders for the normal curve $D = e^{-kz+b}$ and for the standardized curve $D = e^{-kz}$ by integration: $S=\int_a^c e^{-kz+b}dz=\frac{e^b}{k}(e^{-ka}-e^{-kc})$, $S=\int_a^c e^{-kz}dz=\frac{1}{k}(e^{-ka}-e^{-kc})$, where a is the serial number of the first order of branches and b the same for the last order of branches within the range of integration.

EXPERIMENTAL RESULTS

The mean diameters of branches of the same order differed in different corrosion preparations (Tables 1 and 2). Only diameters of branches of the subsequent orders were therefore calculated by the equation $D = e^{-kz+b}$, which was solved by the method of least squares after preliminary taking of normal logarithms in order to find the values of k and b. The number of orders of branches z was found by the equation $z = [\ln(n + 1)/\ln_2] - 3$, where n is the number of alveolar passages determined by the point method. For all cases studied the mean number of orders of branches of the bronchial tree and arterial system was 20.7 ± 0.6.

To study the validity of using the function $D = e^{-kz+b}$ to determine the diameters of branches of the 10th-21st orders, the calculated diameters of branches of the 21st order were compared with measurements made on histological sections. Agreement was good (Table 3).

Experimental study of the rate of change of the mean diameters of the arteries and bronchi confirmed the theoretical arguments. Irrespective of the true diameters of the lung, the rate of change of the mean diameters of the branches was the same (Table 4). Deviation of the theoretically expected rate of change of the diameters (0.243) from the experimentally determined values (average 0.25) can be explained by errors in the calculations and in the measurements, in exact linearity of the experimental semilogarithmic function, etc. Meanwhile, the values of the parameter b were completely determined by the true dimensions of the lung.

TABLE 5. Areas under Exponential Functions $D = e^{-kz+b}$ Characterizing Distribution of Diameters in the Bronchial Tree and Arterial System within Ranges of the 1st-7th, 7th-14th, and 14th-21st Branching Orders

Specimen No.	Area under intervals of exponential curves $D = e^{-kz+b}$					
	1—7		7—14		14—21	
	arteries	bronchi	arteries	bronchi	arteries	bronchi
1	29,1132	36,0520	6,8423	9,0312	1,1783	1,6853
2	26,4799	31,1156	6,3879	7,5265	1,1233	1,2966
3	24,5221	32,7130	5,8507	7,8624	1,0215	1,3545
4	25,7327	33,0390	6,2303	7,9432	1,1054	1,3684
5	24,1172	32,3948	5,4055	7,3887	0,8801	1,2427

After calculation of the parameters k and b and solution of the semilogarithmic equations $\ln D = -kz + b$ the area was determined beneath the exponents $D = e^{-kz+b}$ (Table 5). Fluctuations in the sizes of the corresponding areas for arterial and bronchial corrosion preparations can easily be explained by the different values of the parameter b.

Standardization of the exponential curves $D = e^{-kz+b}$ by making b equal to zero gave curves that coincided completely. Corresponding areas are 2.4201, 0.5743, and 0.0879 mm^2 for the three ranges of branching orders of the arteries and bronchi considered.

The representativeness of the results was verified by the equation for calculation of the sampling size. It was shown that even if the error of the mean rate of change of the diameters was 0.005 (only 2% of the mean), in order to obtain representative conclusions it was sufficient to study three arterial and three bronchial corrosion preparations.

These investigations into the branching of the air passages and the pulmonary arteries thus revealed two distinct groups of parameters: 1) those unconnected with the true dimensions of the lung (the rate of change of the diameters k, the area under the intervals of the standardized exponential function $D = e^{-kz}$); and 2) those determining the overall size of the preparation (the parameter b, numerically equal to the theoretically found logarithms of the diameter of the trachea or trunk of the pulmonary artery, and the areas under intervals of the exponential curve $D = e^{-kz+b}$.

The first group of parameters can be used to assess changes taking place in the lung in pathological states. Investigation of the rate of change of the diameters and the subsequent conversion to areas beneath the intervals of the exponential function are important in topical diagnosis. The second group of parameters is interesting in its own account only in connection with the analysis of fluctuations between the dimensions of the bronchial tree and the arterial system within the species. Meanwhile, it can be applied in order to individualize the results of tests. Determination of the rate of change of the diameters and areas under the intervals of the standardized function and introduction of values of the parameter b allow easy transition to the concrete case. Under these circumstances, the natural logarithm of the diameter of the trachea must be determined for b as a characteristic of the bronchial tree and the natural logarithm of the diameter of the pulmonary trunk for b as a characteristic of the pulmonary arteries.

LITERATURE CITED

1. V. Ebner, in: A. Kölliker, Handbuch der Gewebelehrne des Menschen, Vol. 3, Berlin (1902), p. 292.
2. Hardivillier (1902), cited by E. R. Weibel.
3. K. Matsuba and W. M. Thurlbeck, Am. J. Path., 67, 265 (1972).
4. F. Rohrer, Pflug. Arch. ges. Physiol., 162, 225 (1915).
5. R. Rosen, The Optimality Principle in Biology [Russian translation], Moscow (1969).
6. E. R. Weibel, Morphometry of the Human Lung, Springer-Verlag (1963).
7. T. A. Wilson, Nature, 213, 668 (1967).